1 Increase discourse – increase learning

This book is essentially about increasing the amount of discourse that takes place in a mathematics classroom. There are many reasons this is a good idea, as I will explain as the book continues. However, the chief reason that increasing discourse is important is that it increases the potential for the pupils to learn mathematics and for teachers to help their pupils learn. When you increase the amount of discourse in which the pupils engage, the meaning of words and ideas can be negotiated and extended, and both teacher and pupils can decide on the best way forward with the learning. Once the pupils are engaged in articulating their ideas, Assessment for Learning becomes an embedded part of classroom practice.

I use the word 'discourse' a great deal in this book. By 'discourse' I mean the full range of language use that can be entered into in a classroom. In order to learn mathematics effectively, pupils primarily need to talk about their mathematical ideas, negotiate meanings, discuss ideas and strategies and make mathematical language their own. However, talk is ephemeral and the discipline of writing can make ephemeral thoughts more permanent and, therefore, more easily remembered at a later date. The term 'discourse' also indicates that the pupils are involved in this process; they do the negotiating and discussing, they transform their transient ideas into permanent writing. The teacher initiates and shares in the discourse and manages a process that enables the pupils to become more and more proficient in continuing the discourse. The pupils learn to take part in mathematical discourse and in the process learn to use and control mathematical ideas; they become successful learners of mathematics.

Discourse and Assessment for Learning

Increasing the amount of discourse in a mathematics classroom will increase the use of Assessment for Learning in that classroom. Effective learning and Assessment for Learning – formative assessment – are intimately connected, as has been explained in other publications (Black *et al*. 2002, 2003). If the amount of mathematical language that the pupils use is increased then, among many other benefits, Assessment for Learning can be used effectively, which will, in and of itself, increase the pupils' learning. Therefore, this book is about stimulating increased use of mathematics language by the pupils *and* it is about practical ways to use Assessment for Learning in mathematics, because one leads to the other. Increasing the pupils' ability to use mathematical language means that both the pupils themselves and their teachers can explore their understanding of mathematical concepts and, therefore, either pupil or teacher, or both, will be in a position to act to extend that understanding. It is in acting to extend understanding that the exploration becomes formative assessment and learning is increased.

Mathematical language – a barrier to overcome

Using mathematical language can be a barrier to pupils' learning because of particular requirements and conventions in expressing mathematical ideas. Pupils do need to learn to express their mathematical ideas; teachers cannot expect them to be able to do this without help. For many pupils, learning to use language to express mathematical ideas will be similar to learning to speak a foreign language. Pupils have to learn specific vocabulary, but also means of expression and phrasing that are specifically mathematical and which make it possible to explain mathematical ideas. Unless the pupils know about the way that language is used in mathematics they may think that they do not understand a certain concept when what they cannot do is express the idea in language. Conversely, being able to express their mathematical ideas clearly enables pupils to know that they understand and can use mathematical ideas. Teachers will extend their pupils' ability to learn mathematics by helping them to express their ideas using appropriate language and by recognizing that they need to use language in a way that is different from their everyday use.

Increase mathematical discourse, increase learning

Pupils expressing their own mathematical ideas has many benefits, all of which are intertwined with Assessment for Learning. Once pupils can articulate their ideas they can 'talk through' a problem and can transform the original idea to fit new circumstances. Pupils' ability to articulate their mathematical ideas as they learn enables them to take control of these ideas and transfer them to other situations. They can consider the appropriateness of applying the ideas, try out new ways of using them, take wrong turnings, which they can then assess for themselves, and thereby explore alternative solutions. The ability to talk about ideas gives the pupils the potential to be efficient mathematical problem solvers, and thereby enables them to take on more challenging work. Because the pupils can express their ideas they can control how they use them in ways that tacit learning does not allow.

Pupils and their teacher can become confident of the pupils' understanding when they can express their ideas. Pupils that are able to talk about their mathematical learning can articulate for themselves what else they need to learn. They know what mathematical ideas they can use and can express where to improve their learning. The teacher is able to listen to what the pupils really know, to assess for misunderstandings or for where learning needs to be extended. When the pupils have been taught to use mathematical language to express their ideas the teacher no longer has to 'guess' at the state of the pupils' learning but can act to extend that learning appropriately. Pupils who regularly work to articulate their ideas and understandings can deal with ideas that are usually considered challenging for their age group. They have confidence that they can deal with mathematical ideas and are therefore willing to push at the boundaries of the work they are offered.

Increasing discourse in the classroom will mean that meanings are shared within that classroom. When names are used in mathematics they often convey a complex web of ideas. Consider, for example, the term 'rectangle', indicating a two-dimensional figure with four right angles, four sides and two pairs of parallel lines. Pupils are often asked to consider ideas about the relative length of the sides of a rectangle or that the diagonals cross in the centre of the figure but may not be perpendicular, and so on. A rectangle is a very simple figure but pupils will become aware of an increasing number of associated concepts as they learn more about mathematics. Therefore, the term 'rectangle' will indicate to pupils an increasingly complex web of ideas. When the pupils are involved in the discussion and negotiation meanings are shared. Too often the teacher uses specific mathematical language but the pupils do not. If the pupils do not take a full part in the discourse then they will not 'share' the meaning but instead will have received it, which is not the same thing. Pupils are often reluctant to use mathematical words. Mathematical expressions are not 'their words' but rather words that are used by a community of people that they do not feel part of, and often do not see a way to become part of. It is one of the mathematics teacher's jobs to help her/his pupils bridge this divide. When teachers act to help pupils use essential words and phraseology to express mathematical ideas, they enable the pupils to take part in a learning discourse.

The benefits of pupils' involvement in mathematical discourse

Articulating mathematical ideas

Asking pupils to articulate their current understanding of a mathematical idea enables them to become aware of, develop and reorganize their knowledge. Articulating their ideas helps pupils to remember what they have worked with and makes the knowledge available for them to use and control. They learn mathematical concepts. When pupils articulate their ideas they see themselves as being able to solve mathematical problems.

Being involved in mathematical discourse involves assigning meanings to words and phrases which are shared within a community. If all members of the class take a part in the discourse then everyone shares the meanings generated. Taking a full part in the discourse means that pupils articulate their own ideas, as well as listen to and reflect on ideas that others express. The teacher will also share in these meanings and will therefore have access to pupils' understandings and misunderstandings. The teacher can then modify the teaching activities in the classroom to meet pupils' actual learning needs; that is, they will be able to use formative assessment.

Challenge

When the pupils are engaged in using mathematical discourse more challenging work can be undertaken. Taking part in mathematical discourse enables the pupils to have confidence in what they can do and understand. They know when they have successfully understood concepts and are prepared to use those ideas to solve challenging problems. In order to raise pupils' attainment in mathematics the level of challenge in the work that pupils are asked to do must be as high as possible, without causing them to lose hope of being able to comprehend. Involving pupils in mathematical discourse means that the teacher can be sure that the challenge level is as high possible and the pupils can know that they are learning effectively.

Involving the pupils in the learning process

Discourse enables pupils to be involved in the learning process. This is a primary factor in using Assessment for Learning. When pupils feel involved in the learning process they will be more responsible, more self-efficacious and ultimately more successful. However, to be involved in the learning process in mathematics they must be able to express their ideas and discuss and negotiate with one another; that is, they must be able to use mathematical language.

Part of being involved in the learning process consists of taking some responsibility for the outcome of that process. When pupils take on such responsibility they see the teacher as a resource to help their learning, not as the *only* person who knows what should be done. Discourse in the class can be about the content that the pupils are to learn and also, importantly, about the way that they may learn effectively. When the pupils take a full part in discussions, affecting the course of the discussion and being affected by it, they go away from it able to use the ideas discussed.

Pupils also become involved in the learning process by being offered choice and being allowed, and encouraged, to make decisions about the work that they do and the way that the learning process proceeds. Pupils welcome choice in the way that they continue with their learning, although they may take some time to become accustomed to being allowed to make their own choices. Pupils also become involved in the learning process by being part of the teaching process. When pupils help one another to learn about mathematical ideas they naturally take on the identity of someone who can 'do' mathematics.

Pupils' involvement in the learning process means that they became an integral part of a discourse that develops knowledge; they became part of a meaning-making, discourse community. They can take a meta-cognitive stance, becoming aware of their own learning and beginning to take responsibility for it.

Communities outside the classroom

It is self-evident that the wider community of the school has a great effect on what goes on within the classroom. The effects may be overt – for example, the norms of pupils sitting in rows and not talking during lessons – or subtle – such as pupils' expectations of both their behaviour and the teacher's within a lesson. The pupils may expect not to be involved in the lesson, to have everything organized for them and the teaching 'done to' them. Asking such pupils to think and express their ideas can be a struggle. It would be helpful if the whole school changed its approach and decided to stimulate language use and increased thinking and reflection as part of, say, a drive to improve literacy in the school. However, the lack of a whole-school focus is not a reason to neglect these issues. I know of many mathematics classrooms where pupils articulate and justify their ideas and generate meanings regardless of what happens in other lessons.

Sometimes the whole of society seems to be conspiring against the talking, learning mathematics classroom. Pupils come to mathematics lessons with the idea that there is one right way to solve any mathematics problem and one right answer to that problem. Pupils are often reluctant to give alternative ideas once one has been given. This is understandable as, from their view, all but one answer must be incorrect. These ideas can be overcome in time and with the different approaches advocated in the later chapters. Pupils are often overly concerned about not making mistakes and sometimes would rather do nothing than commit to what might be an erroneous idea. Such feelings stop pupils taking a full part in the discourse. They are reluctant to negotiate or to contribute to a discussion because they are concerned about making a mistake or giving a wrong answer.

Assessment for Learning has a large contribution to make in overcoming this huge barrier to learning in mathematics. First, setting out the learning objectives for the lesson clarifies exactly what and how the pupils are intended to learn. Peer and self-assessment can help build an idea of all the shades and nuances that amount to a high-quality outcome of the learning process. Pupils can use this process to build a confident knowledge that they are 'doing it right' even when their work is different from their peers. Such approaches can help pupils have confidence in their ability to know the required outcome of their work and that they can keep themselves on the right track.

Many pupils come to the classroom with the idea that they have a predetermined and fixed level of ability. In mathematics they are often worried that this level is low. This is an 'entity theory of learning' (Dweck 2000) and is prevalent in much of society. The idea that pupils have a fixed level of ability in mathematics may have been reinforced by 'setting' or 'grouping' procedures in schools, but in other ways as well. The approaches that I am advocating depend on the idea that everyone can become better able to use mathematical ideas by addressing the particular difficulties in learning that they have (the incremental theory of learning). This may be a new idea to the pupils. If, in the past, a pupil had frequently tried and failed to learn mathematics, it is unsurprising if he or she gives up trying. In these circumstances the choice for pupils may seem to be between appearing to be lazy and not trying, or trying and giving the impression of being stupid. It seems, on balance, to be a sensible decision when pupils decide that they would rather be thought lazy than stupid. It is important to emphasize in teaching the incremental view of learning; everyone can improve with perseverance from themselves and help and support from others.

Making connections in mathematics

Increased discourse in the classroom has the potential to help pupils make connections between areas of mathematical learning. In school classrooms, mathematics tends to be taught in a segmented fashion. The lessons are planned under headings: fractions, Pythagoras' theorem, probability, the 'Golf Ball Project', and so on. Pupils will see each set of lessons as quite different from the others unless their teacher takes steps to help them to appreciate the links and connections between them. Mathematics is a series of interconnected ideas; every mathematical area – algebra, geometry, trigonometry, and so on – is part of a whole that constitutes an evolving system, a way of thinking and communicating ideas. Pupils contribute to the system when they generalize or formalize, when they look for patterns or consistency. I would argue that these generic skills – generalizing, searching for patterns, and so on – are ultimately more important to the pupils than, for example, being able to state Pythagoras' theorem, although that would be useful as well. The pupils' view of mathematics tends to be that it consists of patterns and diagrams, and they begin to show an appreciation of symbolic language, explanation, reasoning and justification as being part of mathematics only if this is made explicit. Developing the pupils' ability to take part in the discourse of mathematics enables them to make links and connections across the mathematical system. Pupils begin to see mathematics as a way of explaining, reasoning and justifying, and that the language of mathematics, including non-verbal aspects, has been developed to do this effectively.

Bridges between discourses

Pupils need to make connections, bridges or crossings between their informal discourse and the mathematics register; they are very reluctant to use mathematical vocabulary and phrasing. Lessons using bridging approaches, such as refining the pupils' own attempts to produce a mathematical definition rather than imposing a 'correct' definition, enable pupils to become more adept and more comfortable using mathematical language. Such approaches help the pupils know that they are able to express their own mathematical ideas and are able to use mathematical language. As pupils come to know more vocabulary and are required to express their mathematical concepts more often, they also begin to correct one another in their use of the mathematics register. That is, they begin to make a connection between mathematical language and their own ways of expressing ideas.

It is important to explore the ways the mathematics register fits with, and differs from, the language that pupils use from day to day, otherwise the pupils will be confused and distanced from mathematical ideas. Some pupils find expressing mathematical concepts very difficult and often worry that peers will make fun of them when they try. It is important, therefore, that the classroom ethos recognizes the pupils' difficulties and is supportive and inclusive.

In using mathematical language to explain their ideas, many pupils have to use a discourse that they have not yet made fully their own. It is unsurprising that they feel insecure and open to ridicule when they attempt to act as though they know about mathematics when they do not think of themselves in that way. However, expressing and explaining their ideas helps pupils to learn and to feel that they know them. They take ownership of their ideas and become able to control and use them. This could easily develop into a chicken and egg situation; however, when teachers slowly and carefully nurture pupils' ability to take part in mathematical discourse, they help their pupils to be able to express, and to feel confident about their ability to use, mathematical ideas.

Action research

The main part of this book is about how teachers can act in the classroom to nurture their pupils' ability to use mathematical language so that they can learn effectively. However, first, I will explain why I am so confident that these ideas work and recommend a way of acting that will enable teachers who are seeking to develop their practice to track the changes in their classroom and consider how to make further improvements.

When I started to develop the concepts that this book contains I was a mathematics teacher in an inner-city comprehensive school. The completion of the piece of research (Lee 2004) that underpins the majority of this book led, as it often does, to a change in career that allowed me access to other teachers' classrooms. As they developed practice that reflected my initial ideas I, in turn, learnt more about what it meant to put those ideas into practice. I began to see that the thoughts that had started as an imperative to increase the discourse in my own classroom were powerful in increasing learning because they linked and intertwined with Assessment for Learning. Once the pupils were articulating what they really knew, could do or understand, both I and the pupils could act to increase their learning; I no longer had to guess what would help my pupils – they could tell me. The ideas that started in a small way in my own classroom were tested, proved and extended by many other skilful practitioners that I was lucky enough to work alongside.

I used theory that existed, and that is reviewed in Chapter 6, to attempt to improve my pupils' ability to use mathematical ideas.

> Action research may be defined as '*the study of a social situation with a view to improving the quality of the action within it.*' It aims to feed on practical judgement in concrete situations, and the validity of the 'theories' or hypotheses it generates depends not so much on 'scientific' tests of truth as on their usefulness in helping people to act more intelligently and skilfully.
>
> (Elliott 1991, p. 69, original emphasis)

I knew that I wanted to act more 'intelligently and skilfully' in my classroom and I knew that I had to improve the quality of the learning that was going on. I knew that other authors considered that increasing the discourse in mathematics would increase the learning that pupils were able to do. Now, I thought, how do I increase discourse in my classroom? I tried, and sometimes I failed, and the pupils were confused and irritated by what I asked them to do. But mostly I succeeded a little, and then a little more, as both my pupils and I became more used to what worked and what did not. I found out, by trial and improvement, or, since it was a disciplined study, by action research, how I could support pupils in articulating their own mathematical ideas and I saw that they knew that they learned more this way.

I completed three cycles of action research in order to investigate how I could implement practice that responded better to the ideas that I had acquired through my reading. That is, I planned how to act in the classroom using the theory that I had developed up to that point in time, I implemented those plans and reviewed what I had found out. The implementation and reviewing elements of the action research cycles had three results:

1. a deeper understanding of the various theories that I was reading about
2. a need to search the existing literature to find out more about certain aspects of what I saw happening
3. an expansion or re-articulation of existing theories so that they better reflected the realities of the classroom.

Throughout the three cycles I changed and developed my theoretical perspective. I used my analysis of the data to start to articulate the opportunities and issues of using language in a mathematics classroom and to make this public. The fact that I was a teacher during the data-collection phase of this project was vital to the project and gave strength to its outcomes. I was in a position to create and view the data with a depth of insight given by my intimate involvement in it. My involvement in the 'messy real world of practice' (Griffiths 1990, p. 43) meant that it was difficult at times to collect the data that I needed, but also gave urgency and strength to defining the outcomes. I really wanted to know what my research was indicating as I wanted my practice to be as good as it could be.

The research project

I knew that I wanted to change my practice and I had ideas of how I might go about it; however, I knew that although I would eventually want to use the ideas I was exploring throughout my classes I could not change everything all at once. I picked one class that I got on well with, a Year 9 class (aged 13 and 14 years), with a range of abilities, although none of the pupils felt that they were 'good' at mathematics. I knew that if the ideas worked with this class I could be fairly confident that they would work for most pupils. I also felt that these pupils could achieve much better results in mathematics if I could offer them better learning experiences.

I collected data as I went through the action research cycles. My primary data source was a journal that I kept. I filled in the journal as far as pos-sible after every lesson during the two terms that I researched my practice with this class. In this journal I not only recorded my planning for the class lessons but also detailed thoughts on the pupils' responses to the plans and the way I thought my overall aim of increasing the discourse was progressing. I also used the pupils' notebooks as data and I recorded some lessons so that I could review them later. At the end of the year I conducted informal interviews with groups of pupils where I sought their views about the way that we had interacted within the lessons.

It is hard to collect data when you are teaching; there is so little time. However, I disciplined myself to keep records and I chose ways to record that fitted in with the rest of my work. I made sure that the cycles of the action research fitted in with the terms of the school year, using the breaks for reflection, review and re-planning of the next cycle.

The outcome of the action research cycles

The outcome of the process was for me a surer appreciation of why pupils need to articulate their mathematical ideas, the barriers that prevent them from being able to do so and a series of practices that would enable pupils to develop their ability to express their mathematical thoughts. I also discovered that when pupils started to engage in dialogue in the classroom their learning improved, and both the pupils and I knew what the state of their understanding was and were able to act to increase that understanding.

Action research is a powerful tool to develop teachers' professional practice. The discipline of noticing (Mason 2002) what goes on in a classroom, and reflecting on whether it is as good as it can be, improves the quality of a teacher's own teaching and their ability to share it with others. Action research demands that teachers both think about their own practice and engage with other authors' and colleagues' ideas about what constitutes good practice as they try to improve the quality of their own methods. Action research is a medium through which academic theory can be realized in the classroom.