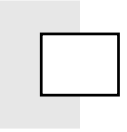

Assessing children's mathematical knowledge

Social class, sex and problem-solving

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comments from various anonymous referees acting for the ESRC and for academic journals.

Chapter 4 is a revised and extended version of a paper initially presented to the Annual Meeting of the American Educational Research Association in New York in 1996, and was published in a shorter form by *Qualitative Studies in Education* in 1998 (reproduced with the kind permission of Taylor & Francis). Chapter 6 is a revised version of a paper originally presented to the Annual Conference of the British Sociological Association in York in 1997, and was subsequently published in a slightly different form in *Sociological Review* early in 1998 (reproduced with the kind permission of the Editorial Board of *Sociological Review*). We would like to thank both journals for allowing us to use this material here. Parts of Chapter 5 were originally presented to the Annual Meeting of the American Educational Research Association in Chicago in 1997.

1 Introduction: background, issues and overview of the book

Changes in school mathematics and its assessment

In recent years, while the research reported in this book has been in progress, school mathematics has been a frequent topic of public discussion. Of course, periods in which school mathematics has been seen as in need of 'improvement' or 'reform' have occurred many times in the past. During such periods the practices of school mathematics are, relative to intervening times, no longer taken-for-granted. They are seen as failing children, or failing employers, or both (Cooper 1985a). The mathematics curriculum and its teachers are attacked by some critics for failing to adhere to the contents or 'standards' of the past (Ball 1994). At the same time, they may be under attack from others for failing to deliver the content and approach to mathematics required by the supposed 'needs' of a predicted future. Such needs are usually read off from some image of recent or emerging economic change (Cooper 1985a). Often, in parallel with these criticisms, there will be links made between school mathematics and broader educational issues, including children's classroom behaviour (e.g. Hayman 1975). It is not difficult to see why mathematics in particular should have been so often a matter of public debate. It has a long history as a high status school subject and, connected with this importance, has played a central role in the selection of children for higher education by means of public examinations. Children's supposed levels of intelligence have often been inferred from their attainments in school mathematics. Furthermore, the subject has been seen to be, and in many ways is, highly relevant to a variety of occupational roles, of lower and higher social status (e.g. Cockcroft 1982; but see Dowling 1998, for a critique of this view). It has also been the case, certainly until the relatively recent past, that the subject has been understood by most interested individuals and organizations as one in which correctness and incorrectness,

and, via these, children's knowledge and understanding, are easy to identify. This belief has allowed the development of a testing industry in mathematics and the use of the products of this industry to make supposedly valid comparisons between individuals, schools and even nations (Stake 1992). In the case of England and Wales, in the past decade, there has been politically-driven reform of curriculum content, pedagogy and assessment resulting in a programme of national testing of mathematics, English language and science. The mass media now engage in an annual frenzy of school and district comparison, drawing on statistics of performance deriving from the administration of tests in these three subject areas at several points in a child's school career, and from examinations in a broader range of subjects at age sixteen. It is most often teachers, schools and children who bear the brunt of any resulting criticism, with little account being taken of any contextual factors. The tests themselves receive criticism only in professional and academic arenas.

In this book, we wish to redress the balance a little, by focusing on the various ways in which paper and pencil tests of mathematics may not reflect adequately what children actually know and understand of the subject. Researchers have shown, in recent years, that both adults and children are able to undertake arithmetical calculations outside of school, employing a variety of approaches not taught in school, even though they are apparently unable to produce similar results when faced with school problems and are required to use school-taught algorithms (Scribner 1984; Nunes *et al*. 1993). Our focus is a related one, but different in emphasis. Since the reaction against the abstract algebraic approaches favoured by the curriculum reformers of the 1960s, many influential voices working within mathematics education, whether within para-governmental institutions or within institutions producing future teachers, have favoured an approach to mathematics teaching and learning which has as a key focus the relating of work in mathematics to some version of the 'real world' (e.g. Cockcroft 1982; NCTM 1991). This approach is supported for a variety of reasons. For some, the main justification is motivational, based on a belief that children will find mathematics more interesting and relevant to their concerns if, for example, techniques of calculation are taught in the context of consumption or work – or, at least, via textual representations of such contexts. For others, the main motivation derives from the supposed needs of children as future consumers and workers. In England, much of the argument for a more 'relevant' and 'realistic' approach was disseminated to the world of education at large in the Cockcroft report (1982) on school mathematics. Subsequently, the Education Act of 1988 introduced a National Curriculum (NC) in England and Wales and, alongside this, a programme of national assessment of children's attainments at the ages of 7, 11, 14 and 16. This programme of assessment has comprised elements internal and external to the school,

but, as a result of both political debate and practical experience of the original policy in practice, their relative importance has shifted (Cooper 1998b; Dunne 1998). As a result, by the middle of the 1990s, the main emphasis had moved from one focusing on assessment of children's work by teachers as part of their everyday classroom activities, to one focusing on assessment via the use of nationally designed paper and pencil tests marked by personnel external to the school in which the test was to be administered. The NC assessment tools were designed, under contract, mainly by university personnel previously experienced in mathematics education and who, as a result, seem to have carried with them, as one taken-for-granted assumption, the belief that 'relevance' and 'realism' were important aspects of educational practice in mathematics. As the 1990s progressed the designers of the assessment packages found themselves under increasing pressure to simplify the assessment tools and, in particular, to make more use of externally marked timed paper and pencil tests. This conjunction of factors led to tests being produced that consisted predominantly of items embedding mathematical operations in textually represented 'real life' situations.

Such 'realistic' test items can cause a variety of problems for children. They often require a greater amount of reading compared with 'non-realistic' items before the mathematical point can be addressed, which might be expected to disadvantage poorer readers. It has also been shown, in several research studies, that children frequently fail to apply 'realistic' considerations when it is defined as 'appropriate' that they do so by test designers (e.g. Verschaffel *et al.* 1994). (We discuss some of the characteristics of these studies in Chapter 3.) However, there is another potential problem generated for children by 'realistic' items, and one that has received much less attention in the research literature. Children, and especially those children who are less knowledgeable about the peculiar ways boundaries are drawn between school and everyday knowledge, will perhaps fail to demonstrate what they know and understand about mathematics as a result of drawing 'inappropriately', from the perspective of the test designers, on their 'everyday' knowledge of the world outside of the classroom (Cooper 1992, 1994b). This is one problem, alongside others, with which this book is concerned. In reporting the ways in which children cope with this 'boundary' problem, we pay attention to potential differences between boys and girls, and between children from different socio-economic backgrounds. We explain why below.

Research in mathematics education

Much research on children's understanding of mathematics has been carried out without reference to children's cultural backgrounds, often

by psychologists or others using psychological frameworks of analysis. Notwithstanding the important tradition of work in cultural psychology (e.g. Cole *et al.* 1971; Cole 1996), revivified in recent years as a result of a renewed interest in the work of Vygostsky (e.g. Cobb 1994; Wertsch 1995), many researchers seem to have bracketed out extra-school cultural considerations in their approach to the study of children's problem-solving in school mathematics. As a consequence research findings are often presented in terms of some supposedly typical child, though, increasingly, results may be broken down by sex, either because of concerns with equality of opportunity or because of beliefs concerning sex differences in cognitive predispositions (e.g. Fennema *et al.* 1998). In the past, the lack of attention to culture may have resulted partly from a working assumption that children's cognitive development followed similar patterns everywhere. But, in the case of mathematics, it may also have been partly dependent on various tacit assumptions about mathematics itself, including assumptions concerning the universality of a discipline that many appear to believe to be above and beyond 'everyday' cultural differences. However, there are several reasons for believing that a consideration of cultural differences between children might allow us greater understanding of their success and failure in problem-solving in school mathematics.

First of all, it is important to note that mathematics, and school mathematics in particular, is not unchanging. Clearly, the content created and studied by university research mathematicians will change over time, and such changes will, via what Bernstein (1996) has termed recontextualizing processes, sometimes be reflected in changes in the content of school mathematics, though often only after a considerable time lag and after periods of conflict within the disciplinary group (Cooper 1983, 1985a). But, as we have already indicated, much of the change at school level only partly concerns the underlying mathematical operations themselves. Change in school mathematics is often about different matters. For example, around the period of World War II in England, a key focus of debate was the boundary between algebra, trigonometry and geometry with some individuals arguing for a more 'integrated' approach to the teaching of these three elements of the curriculum in the selective secondary schools. This desire to 'integrate' returned as part of the debate in the 1960s, where it became linked more explicitly with the wish of some reformers to introduce some modern abstract algebra into school courses. At the same time, some argued for a greater emphasis on 'realistic' applications of mathematics, wanting a weakening of the boundary between mathematics and 'everyday' and 'economic' concerns (Cooper 1985a). This demand appeared again in the early 1980s. There has also been an on-going debate over pedagogy between, at the extremes, those favouring the learning of algorithms and those favouring children's discovery and/or invention of mathematics for themselves (Ernest 1991).

This latter debate often seems to be driven as much by preferences concerning the supposed respective merits of established authority as opposed to popular democracy as by the nature of mathematical knowledge or the technical merits of various learning theories.

If we consider the elements of these debates – 'boundaries' within mathematics and between mathematics and 'everyday' matters, 'realism' with its associated question of whose 'everyday' concerns are to be represented in school mathematics, and the debate between supposedly authoritarian and democratic pedagogic approaches – we can see that the distinction between mathematics on the one hand and cultural matters on the other is not easy to draw. Indeed, 'culture' and associated ideas such as 'communities of practice' have begun in recent years to receive more attention within the field of mathematical learning and problem-solving. Some of this discussion has employed a rather restricted use of the 'culture' concept in which it is used to capture what are seen as defining elements in particular classroom approaches to mathematics or what it is that a teacher and his or her pupils share as a result of their work together (Davis 1989). Some of it, however, moves beyond this narrow concern to consider extra-school culture in relation to mathematical problem-solving, broadly conceived (e.g. Lave 1988). There has also been a growing literature on the relations between mathematics and its cultural context (e.g. MacKenzie 1981). There has been increasing discussion of ethnicity and mathematics (Shan and Bailey 1991) and, informed by feminist thinking, there have been claims that school mathematics typically has been 'gendered' in ways that favour boys over girls (e.g. Walkerdine *et al.* 1989). The latter claim, of course, only makes sense on the basis of assumptions about the ways in which boys and girls typically differ from each other, either in terms of cognitive predisposition or cultural identity. Turning to this side of the relation – the child – we can note that there has been a long tradition of analysing cultural differences between social groups with particular reference to their relevance for school achievement. Orientations to time, linguistic resources, orientations concerning 'abstraction', willingness to tolerate ambiguity, 'cultural capital' and many other factors have been discussed in the literature. Writers addressing these matters have often been attacked for characterizing the members of 'disadvantaged' groups in terms of a 'deficit', read off by comparing them with some other group presented as 'normal'. We would readily agree that sometimes this criticism is justified, but if carried too far it rules out a quite proper sociological concern with the consequences of the *relations* between a child's cultural resources, which are given on entry to the school, and what the school demands of the child as the conditions of his or her success. For a simple example, consider the child whose domestic language is 'B' but whose school demands the use of language 'A'. This child is not in any intrinsic sense linguistically deprived,

but clearly is *relatively* deprived of a resource that the school demands for 'normal' progress to occur. It is true that the child's relative deprivation could be removed by a change in the school's linguistic medium of teaching and learning, though such a change might, of course, leave the child relatively deprived at a later stage of life. The key point here, however, is that, in the absence of such a change, the child is likely to do less well at school than speakers of language 'A', all other things being equal. Recent debates in California over the language of teaching, learning and assessment for pupils of 'limited English proficiency' illustrate the dilemmas that exist in this area.

There has been a neglect, especially within research on mathematics education, of the ways in which cultural differences between children from differing social classes might influence their success and failure in mathematics (Apple 1992, 1995a, 1995b). This is partly a result of a reaction against what were seen as 'deficit' theories and partly because of a relative falling away of concern with social class differences in educational achievement in comparison with the post-war period. Much more energy has been expended on the important areas of gender and ethnicity. But a little reflection on the existing literature on social class differences in attitudes to formal knowledge and problem-solving suggests that this area deserves further attention. For example, both Bernstein (e.g. 1996) in England and Bourdieu (e.g. 1986) in France have produced evidence which suggests, respectively, that both children and adults from the dominant and subordinate social classes of these societies differ considerably in their orientation to the boundary between 'everyday' and 'esoteric' knowledge. We return to their work in Chapter 4, but here we can just note the obvious potential relevance of such research to 'realistic' school mathematics. On the working assumption that their past research remains of relevance in more recent times, we can see that a shift to a curriculum and to assessment items that embed mathematical operations in 'everyday' contexts might not be neutral in its effects with respect to children from different social class backgrounds. We will show that it is possible that assessment via 'realistic' items might serve to exaggerate reported differences in mathematical understanding between the social classes. Furthermore, even if the details of these particular sociologists' claims were to be found wanting, either because of problems in their initial work or because of recent social and cultural change, the general point remains important. Changes in school subjects, and in assessment regimes in particular, should be examined not only from the point of view of their technical adequacy, that is their validity and reliability, but also in terms of the ways their demands intersect with the cultural resources of the various social groups confronted with them.

There a number of points we should add here, before describing the organization of this book. We know already, as a result of responses to

our work at conferences, that there will be a few individuals within the world of mathematics education who will wish that we had not produced this account of the difficulties children experience with 'realistic' items. Such critics may be variously motivated. At worst, they may have a commitment to the merits of 'relevant' and 'realistic' school mathematics, which brooks no criticism. More acceptably, they may fear that those demanding a return to a regime emphasizing the rote-learning and testing of simple arithmetical algorithms may find support in the work reported here. We have some sympathy with this concern. However, practices in education must be considered in terms of a number of values simultaneously. Such values, for us, alongside those relating to pedagogy itself, include those relating to the fairness of assessment processes as well as to their technical adequacy. And, while we obviously have our own preferences concerning the nature of school mathematics, it remains crucially important, whatever current practice is, that any unintended effects it might have, with respect to either cognitive development or social selection, should be analysed with reference to evidence rather than personal preference. It may seem obvious to some, for example, that working-class children will benefit from 'relevant' or 'realistic' school mathematics, but perhaps we should check. Furthermore, even accepting that there may be good pedagogic or economic reasons for employing 'realistic' approaches in school mathematics, we still have much to learn about the ways in which their introduction, especially in the context of assessment, operates to mislead us about *which* children know exactly *what* mathematics. We hope this book proves a useful contribution to this understanding.

Organization of the book

Chapter 2 begins with a brief account of the origins and nature of the assessment regime for school mathematics in England at the time of our research. It then describes the research project on which this book is based. It ends with a discussion of two problematic variables, 'ability' and social class. Chapter 3 begins with a little history, looking at various claims concerning the relations between the 'abstractness' of mathematics and problems in the 'real world'. It then offers a critical account of some previous research on children's approaches to 'realistic' mathematics problems, before ending with an analysis of some of the peculiarities of National Curriculum test items and a brief discussion of one possible way of understanding children's responses to them. In Chapter 4, we employ the responses of two children to several 'realistic' test items in order (1) to illustrate the distinctive ways in which children might respond to such items, and (2) to introduce some key ideas from the work of Bernstein and Bourdieu on the manner in which individuals

from different social class backgrounds might vary in their response to problem-solving situations. The chapter ends by providing a model of the ways in which culture might interact with assessment to produce 'differential validity'. In Chapter 5 we present two main statistical analyses of children's mathematical test performance at 10–11 years of age. Initially, after considering some technical problems concerning the reporting of achievement by NC 'levels', we present an overall analysis of performance by social class, sex, school and measured 'ability'. We then consider the ways in which children from various socio-cultural backgrounds perform on the two separate subclasses of items, which we have termed 'esoteric' and 'realistic'. In Chapter 6 we explore the responses of more than one hundred children to just two of the 'realistic' items previously discussed in Chapter 4, with two purposes. First, we wish to examine whether the possible social class differences in children's modes of responses pointed to in Chapter 4 exist across our sample as a whole. Second, we wish to consider whether some part of the social class differences in relative performance on 'esoteric' and 'realistic' items reported in Chapter 5 can be accounted for by the differentiated ways in which children interpret 'realistic' items. In Chapter 7 we explore the relationships between the overall achievement of 13–14-year-olds in the NC tests and a variety of factors. The chapter begins with an analysis of the distribution of children by our research schools to the hierarchically organized tiers of the NC tests. It considers the possible effects of the patterning of this distribution on measured achievement in the NC tests. The chapter then considers NC achievement in relation to sex, social class, school and measured 'ability'. In Chapter 8 we return to explore children's struggle with the meaning and demands of particular items. We begin with an analysis of the responses of both 10–11- and 13–14-year-olds to a data-handling item, which originally appeared in the Key Stage 2 tests. We then move to analyse the responses of higher attaining 13–14-year-olds to an item concerning probability. Our focus in this chapter is on the apparent tendency of some children to produce inadequately explicit responses to items requiring elaborated 'reasons' to be given for answers, and the consequences of this tendency for the validity of the assessment of these children's knowledge and understanding. In Chapter 9 we compare children's responses to two Key Stage 3 algebra items, which involve the constructing and simplifying of expressions. Our focus is on the ways in which the embedding of one of these tasks in an 'everyday' context leads to difficulties for some children in understanding the intended goal of the problem. We also consider the ways in which such difficulties are distributed by social class and sex. We turn now, in Chapter 2, to a description of our research programme.